hw set 21
sect 8.2
p574

4. $\sum_{n=1}^{\infty} \frac{2n^2-1}{n^2+1}$

$S_1 = \frac{1}{2}$

$S_2 = \frac{1}{2} + \frac{7}{5} = 19/10$

$S_3 = S_2 + 17/10 = 18/5$

$S_4 = S_3 + 31/17 = 461/85$

$S_5 = S_4 + 49/26 = \frac{16151}{2210}$

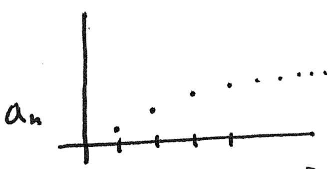
$S_6 = 9.2271$

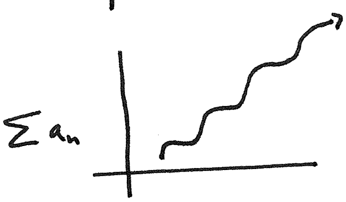
$S_7 = 11.1671$

$S_8 = 13.1209$

$S_9 = 15.0843$

$S_{10} = 17.0546$





diverges b/c Sum $\to \infty$

11. $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \ldots$

$5 \cdot 1 - 5 \cdot \frac{2}{3} + 5 \cdot 4/9 - 5 \cdot 8/27 + \ldots$

$= \sum_{n=1}^{\infty} 5\left(-\frac{2}{3}\right)^{n-1}$ $\quad a=5,\ r=-2/3$

$= \frac{5}{1-(-2/3)} = \frac{5}{5/3} = 3$

21. $\sum_{n=1}^{\infty} \frac{1+2^n}{3^n} = \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n + \left(\frac{2}{3}\right)^n$

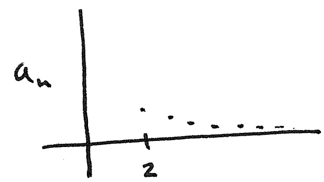
$\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n = \sum_{n=1}^{\infty} \frac{1}{3}\left(\frac{1}{3}\right)^{n-1} = 1/2$

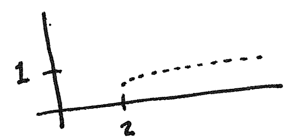
$\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n = \sum_{n=1}^{\infty} \frac{2}{3}\left(2/3\right)^{n-1} = 2$

$\sum_{n=1}^{\infty} \frac{1+2^n}{3^n} = 2\frac{1}{2}$

8. $\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$

$S_2 = 1/2$		$S_7 = 6/7$	
$S_3 = 2/3$		$S_8 = 7/8$	
$S_4 = 3/4$		$S_9 = 8/9$	
$S_5 = 4/5$		$S_{10} = 9/10$	
$S_6 = 5/6$		$S_{11} = 10/11$	





converges since $\forall\ S_n = \frac{n-1}{n} < 1$

12. $1 + .4 + .16 + .064$

$= \sum_{n=1}^{\infty} (.4)^{n-1} = \frac{1}{1-.4} = \frac{1}{.6} = 1.\overline{6}$

13. $\sum_{n=1}^{\infty} 5(2/3)^{n-1} = \frac{5}{1-2/3} = \frac{5}{1/3} = 15$

18. $\sum_{n=1}^{\infty} \frac{n+1}{2n-3}$ diverges since $\frac{n+1}{2n-3} \longrightarrow \frac{1}{2} \neq 0$

28. $\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3} = \sum \frac{2}{(n+1)(n+3)} = \sum \frac{1}{(n+1)} - \frac{1}{(n+3)}$

$S_n = \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{4}} - \frac{1}{6}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}\right) + \ldots$

$+ \left(\cancel{\frac{1}{n}} - \frac{1}{n+2}\right) + \left(\cancel{\frac{1}{n+1}} - \frac{1}{n+3}\right)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3} = \frac{5n^2+13n}{6n^2+24n+18}$

$S_n \longrightarrow 5/6$

32. $.\overline{73} = .737373\ldots$

$\frac{73}{99}$

34. $6.2\overline{54} = 6.2545454\ldots$

$$\begin{aligned} 100x &= 625.45454\ldots \\ -\quad x &= 6.25454\ldots \\ \hline 99x &= 619.2 \end{aligned}$$

$$x = \frac{6192}{990} = \frac{344}{55}$$

35. $\sum_{n=1}^{\infty} \frac{x^n}{3^n} = \sum_{n=1}^{\infty} \left(\frac{x}{3}\right)^n$ geometric

if $\left|\frac{x}{3}\right| < 1$ converges

$-1 < \frac{x}{3} < 1$

$-3 < x < 3$

36. $\sum_{n=1}^{\infty} 2^n (x+1)^n$ geometric

$= \sum (2x+2)^n$

if $|2x+2| < 1$ converges

$-1 < 2x+2 < 1$

$-3 < 2x < 1$

$-\frac{3}{2} < x < \frac{1}{2}$

54. EC $f_1 = 1,\ f_2 = 1,\ f_n = f_{n-1} + f_{n-2}$

a) $\frac{1}{f_{n-1} f_{n+1}} = \frac{1}{f_{n-1} f_n} - \frac{1}{f_n f_{n+1}}$

$$\frac{1}{f_{n-1} f_n} - \frac{1}{f_n f_{n+1}} = \frac{f_{n+1} - f_{n-1}}{f_{n-1} f_n f_{n+1}}$$

$$= \frac{f_n}{f_{n-1} f_n f_{n+1}} = \frac{1}{f_{n-1} f_{n+1}}$$

b) $\sum_{n=2}^{\infty} \frac{1}{f_{n-1} f_{n+1}} = 1$

$$\sum_{n=2}^{\infty} \frac{1}{f_{n-1} f_{n+1}} = \sum_{n=2}^{\infty} \left(\frac{1}{f_{n-1} f_n} - \frac{1}{f_n f_{n+1}}\right)$$

$$= \left(\frac{1}{f_1 f_2} - \cancel{\frac{1}{f_2 f_3}}\right) + \left(\cancel{\frac{1}{f_2 f_3}} - \cancel{\frac{1}{f_3 f_4}}\right) + \ldots$$

$$= \frac{1}{f_1 f_2} = \frac{1}{1 \cdot 1} = 1$$

c) $\sum_{n=2}^{\infty} \frac{f_n}{f_{n-1} f_{n+1}} = 2$

55. EC